Solution of fuzzy elliptic PDEs by a polynomial chaos expansion

Samuel Corveleyn

Department of Computer Science, K.U.Leuven
Celestijnenlaan 200A - bus 2402, 3001 Heverlee
Belgium
samuel.corveleyn@cs.kuleuven.be

Joint work with: Stefan Vandewalle

Mathematical models of engineering problems often contain parameters whose values are uncertain. Uncertainty is usually modeled with random numbers or fields. Simulations become unreliable however if the precise probability distribution of the random parameters is not well known. In that case, fuzzy numbers and fields may be a safer choice to model uncertainty. They allow a more reliable analyis with only little information about the characteristics of the uncertainty.

We introduce a new approach to solve fuzzy elliptic partial differential equations of the kind

$$-\nabla \cdot (\widetilde{a}(\mathbf{x})\nabla \widetilde{u}(\mathbf{x})) = f(\mathbf{x}),$$

with $u(\mathbf{x})$ the unknown and $\widetilde{a}(\mathbf{x}) = a_0(\mathbf{x}) + \sum_{r=1}^{N_\xi} \widetilde{\xi}_r a_r(\mathbf{x})$ a fuzzy field diffusion coefficient. In its parameterized formulation, the equation becomes

$$-\nabla \cdot (a(\mathbf{x},\boldsymbol{\xi})\nabla u(\mathbf{x},\boldsymbol{\xi})) = f(\mathbf{x}).$$

Solving the fuzzy equation comes down to a global optimization of $u(\mathbf{x}_i,\boldsymbol{\xi})$ over different α-level sets of the uncertain input parameter $\widetilde{\boldsymbol{\xi}}$ for a set of points $\{x_i\}_{i=1}^{N_x}$. Solving this large set of optimization problems might get very expensive because of the PDE constraint. Response surfaces are therefore often used as a surrogate function because they are cheaper to evaluate. Currently, black-box approaches to construct the response surface, like Kriging, are used to solve fuzzy differential equations. They however lack proof of accuracy and convergence. We propose to use methods from stochastic PDE theory, based on polynomial chaos expansions. They have been shown to have excellent convergence properties. We proof that these convergence results imply convergence of the approximate fuzzy solution to the exact fuzzy solution.

Keywords: Approximation and interpolation, Partial differential equations, Uncertainty, Fuzzy, Polynomial Chaos.